Fluid-particle Heat Transfer in Packed Beds

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Steady state heat transfer experiments were carried out in a 4-in. I.D. transite tube packed with 3/8-, 1/4-, and 5/32-in. steel spheres. Heat was generated in the pellets by means of a high-frequency induction coil surrounding the test section. Average heat transfer coefficients between the bed of spheres and a stream of air passing through the bed were calculated for Reynolds numbers of from 200 to 10,400. To ensure the reproducibility of the data, the bed was repacked six times for each pellet size.

A study of the effect of the tube-to-pellet-diameter ratio indicates that this effect is large for low values of the ratio, but much smaller for higher ratios. The results are presented both graphically and in terms of empirical equations. The analogies among heat, mass, and momentum transfer are discussed, and it was found that no simple relation between the heat transfer coefficient and the friction factor exists for packed beds with a gas as the fluid.

An attempt is made to predict the heat transfer rates for packed beds from heat transfer data for single spheres and from pressure-drop measurements for the packed bed; however, the rates predicted from the pressure-drop measurements are somewhat lower than the experimental results.

Packed beds have been used extensively in industry as catalytic reactors and also for heat transfer, absorption, and adsorption operations. Although many investigations have already been carried out to determine the heat transfer characteristics of packed beds, the direct application of the basic relationships developed to the design of packed beds has been held back by the inconsistancy of the data which have appeared in the literature.

Previous investigations have been carried out by both steady and unsteady state methods. In the steady state investigations the heat transfer coefficients are determined from a knowledge of the amount of heat transferred and the temperature difference between the pellets and the gas. In the unsteady state measurements the temperature history of the pellets and gas is measured and the coefficients are determined by a comparison of these data with the theoretical curves developed by Schumann (*17*). These curves were obtained from a solution of the differential equations describing the heat transfer, in a bed of solids with no heat loss through the walls. Brinkley (*3*) extended the mathematical treatment to the case in which the solid generated heat. By using the concept of the effective thermal conductivity, Amundson (*1*) extended the treatment to include almost every conceivable case.

For the present investigation it was decided that an accurate investigation of the heat transfer relationships in packed beds could be obtained from a steady state system in which heat could be generated in the particles themselves at a sufficient rate to produce a large temperature difference between the particles and the gas passing through the bed. It was found that this could be accomplished by the use of a large high-frequency induction unit if metallic particles were used in a nonmetallic tube. Spheres were selected as particles so that the exact surface area would be known and also so that the data might be more readily interpreted for this regular packing.

It has been found that the ratio of tube to particle diameter has an effect on the heat transfer coefficient (*4*); therefore, three particle sizes were used, which allowed the ratio to vary from about 10 to 25. A bed diameter of 4 in. was selected so that pellets might be used which were large enough to permit the insertion of thermocouples without too great an effect. A bed height of 4 in. was used in order to decrease the end effects which might occur with a shorter bed.

As a bed is usually packed at random, the particles assume a different arrangement each time the bed is packed. In order to obtain data which would be

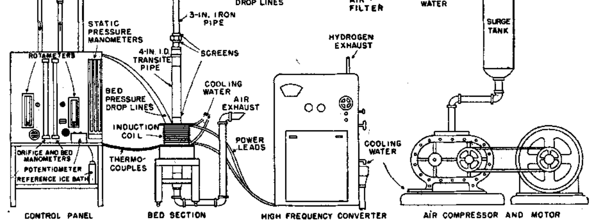

Fig. 1. Equipment layout.

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reproducible, the bed of spheres was emptied and repacked six times for each set of runs.

In order to interrelate heat and momentum transfer in the bed, pressure-drop measurements were also taken for each run.

EXPERIMENTAL APPARATUS AND PROCEDURE

Figure 1 shows a diagram of the experimental apparatus used in this investigation. Air, supplied by a blower, was passed through a cooler and a flow-metering instrument before entering the test section. The air entered the bed from the top and was discharged to the atmosphere after it passed through the bed. The cycloidal blower was designed to deliver 300 cu. ft. of air/min. when it was discharged at 15 lb./sq. in. gauge. The piping system was constructed of standard 2-, 3-, and 4-in. steel pipe. All valves used in the system were gate valves, to reduce the pressure drop through the system. The flow rates were measured by rotameters in the low flow range, and by an orifice for the higher flow rates.

The section containing the bed was constructed from a 4-in. I.D. Transite vent pipe ½ in. thick and 5 ft. long. This section was glued with an adhesive into sleeves welded to the steel-piping system. Figure 2 shows a detailed drawing of the packed section of the tube. The 4-in.–high test section was equipped with Bakelite flanges to facilitate removal and replacement of the packing. The packing support, which was constructed from a ½-in. wood fiber plate, consisted of a number of slits $\frac{3}{32}$ in. wide and spaced at distances of $\frac{3}{16}$ in. from center to center. Rubber gaskets were used with the flanges, and the bed was held in place by bolts machined from wood fiber. Pressure taps were drilled into the flanges at the entrance and exit of the bed on opposite sides of the flanges. An adhesive was used to connect the various parts of the bed, and glass wool insulated the bed and a section 3 in. above and below it.

Three different packings, 0.1555-, 0.2495-, and 0.3745-in.–diameter high-carbon-chrome–alloy steel ball bearings, were used in this investigation. Copper-constantan 30-gauge thermocouples were used throughout, and the thermocouples in the neighborhood of the induction coil were twisted in order to cancel the effect of the magnetic field on the generated electromotive force. No effect was observed on the thermocouple readings because of the electrical field. The thermocouples which measured pellet temperatures were placed in $\frac{1}{32}$-in. holes drilled into the center of the pellets, and the holes were filled with solder. At the bottom of the bed the pellets containing the thermocouples were spaced across the diameter of the bed so that the bottom layer of packing assumed a regular arrangement. The thermocouples measuring the air temperature at the exit of the bed were glued to the packing support directly below these pellets. At the top of the bed the pellets containing the thermocouples were evenly spaced across the diameter of the bed by means of a row of pellets glued together and to the tube wall. Thermocouples were also attached to the outside tube wall to obtain the heat loss through

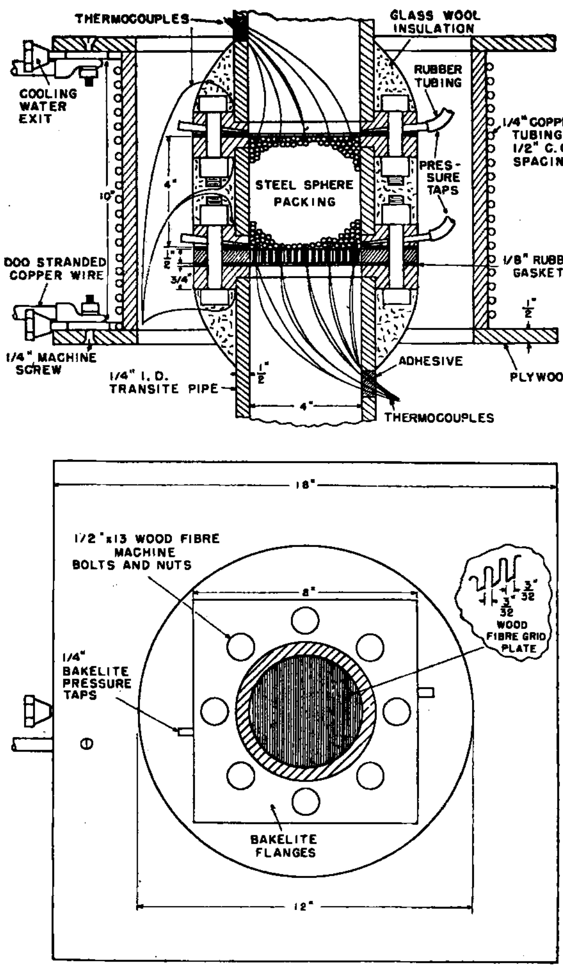

Fig. 2. Detail of bed and induction coil.

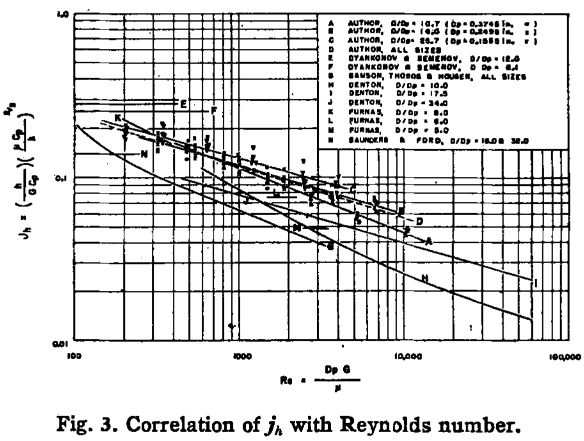

Fig. 3. Correlation of j_h with Reynolds number.

the wall. The air temperature was measured at five positions across the diameter at the exit of the bed, and since the air entered at a uniform temperature, only one temperature was required at the entrance of the bed. The pellet temperatures were measured at five positions across the diameter of the bed at both the entrance and exit of the bed. Suitable corrections, always very small, were made for the effects of radiation. The thermocouples used in the investigation were insulated up to the junctions. The maximum error due to conduction along the thermocouple leads was calculated to be less than 2%, which is well within the experimental error.

The induction coil used was 13 in. in diameter and 10 in. long and was constructed of ¼-in. copper tubing with ½-in. center-to-center spacing. The high-frequency current for the coil was supplied by a 20 kw. Ajax-Northrorp converter. The spark gap of the converter operated in an atmosphere of hydrogen and produced current having a frequency of about 20,000 cycles.

The packing of the bed was carried out by placing the first two layers of pellets in the bed by hand, forming a regular packing. The rest of the bed was then filled by allowing the pellets to fall in place randomly from a stream of pellets at the center of the bed. The last layer of the bed was left out while the row of pellets containing the thermocouples was allowed to dry. Pellets were then filled in around this line of pellets to form a constant level at the top of the bed.

The air flow rate through the bed was controlled by an entrance bleed-off valve and by the flow-meter valves. Cooling water was used in the induction coil, the blower, and the high-frequency converter. The operating level of the converter was increased with increasing flow rate, and for each packing of the bed six different flow rates were used, the bed being repacked six times for each pellet size. For each run sufficient time was allowed for the temperatures in the bed to assume a steady state distribution. In addition to the temperature measurements, the static pressure at various points in the system and the pressure drop through the bed were also measured, and it was found that negligible pressure drop occurred across the empty bed.

EXPERIMENTAL RESULTS

The average heat transfer coefficients for the bed were calculated from the total heat generated in the pellets, the surface area of the pellets, and the mean temperature difference between the air stream and the pellets. The total heat generated by the pellets was determined as the sum of the sensible heat gain of the air passing through the bed and the heat loss through the wall. The latter was found to be quite small in comparison with the sensible heat. The heat transfer coefficient at the wall used to obtain the wall loss was obtained from the data of Leva (*13*). The mean driving force was calculated as the log mean temperature difference of the average driving force at the entrance and exit of the bed. The mean driving forces obtained in this investigation varied from about 4° to 30°F., depending on the flow rate and pellet size. The average driving forces at the entrance and exit of the bed were obtained from a graphical integration of the radial temperature differences versus the square of the radius. This produced an area mean driving force for the entire cross section. The data taken are too numerous to present here and may be found in reference *2*. A typical run is given in Table 1.

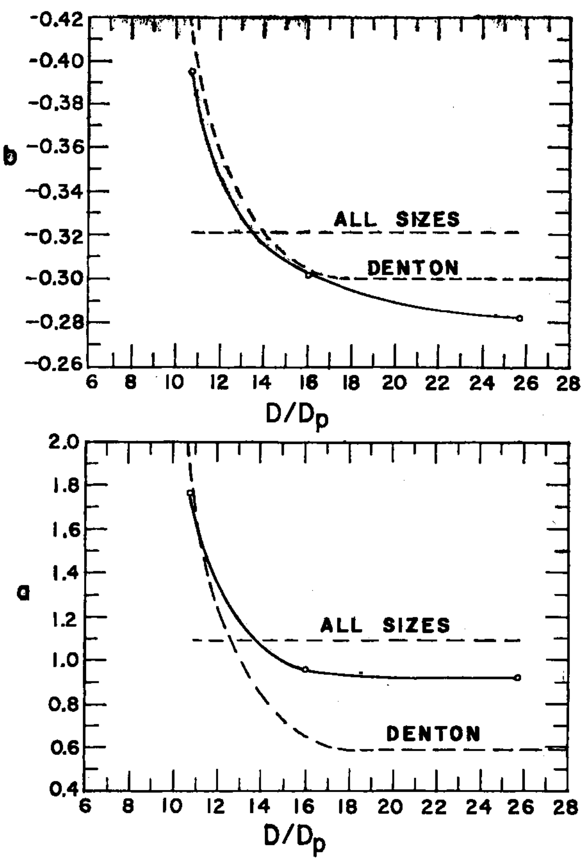

Fig. 4. Coefficients for j_h correlation.

The inlet air temperature was measured above the top layer of pellets and the outlet air temperatures were measured below the bottom layer of pellets. The air temperatures corresponding to the pellet temperatures in each layer were obtained from the longitudinal air temperature gradient.

The average heat transfer coefficients obtained in this investigation are shown in Figure 3 in terms of j factors, along with the results of other investigators. The physical properties used in each case were taken at the arithmetic average temperature and pressure of the air between the entrance and exit of the bed. The air entered the bed at room temperature and attained temperatures up to 300°F. passing through the bed.

These data show a significant difference for each pellet size. Whereas the data for a given packing of the bed were found to be reproducible, the scattering of the

TABLE 1

EXPERIMENTAL DATA

Run 5B2
Pellet diameter = 0.2495 in.
Number of pellets = 3,996
Void fraction = 0.354
Total heat generation = 5,237 B.t.u./hr.
Mass velocity = 1,270 lb./(hr.)(sq. ft.)

	Outlet temp., °F.			Inlet temp., °F.	
r/r_0	Air	Pellet	r/r_0	Air	Pellet
0.843	280.9	293.9	0.750		117.9
0.469	277.0	290.0	0.375		105.8
0.000	258.6	271.6	0.000	89.1	106.3
0.463	275.3	288.3	0.375		104.4
0.843	283.1	296.1	0.750		119.2

data for a given pellet size is due to repacking the bed six times. Since a log-log plot of the data produced a straight line for the entire range studied, the data were correlated as

$$j_h = a\, Re^b \qquad (1)$$

A separate equation was obtained for each particle size as well as an equation which represented all particle sizes. These are listed in Table 2, and are somewhat

TABLE 2

CORRELATION OF j_h WITH REYNOLDS NUMBER

Pellet size, in.	a	b	95% Confidence limits, %
0.3745	1.58	−0.40	15.6
0.2495	0.96	−0.30	20.1
0.1555	0.92	−0.28	24.9
All sizes	1.09	−0.32	30.4

higher than those found by Gamson et al. (*10*), Denton (*4*), and Saunders et al. (*16*) and lower than those found by Dyankonov et al. (*5*). The experiments of Furnas (*9*) and of Saunders (curves *LMN*) were carried out under unsteady state conditions, and the Schumann curves were used to determine the heat transfer coefficients. In each case the heat was transferred from a hot gas to a bed of particles initially at a lower temperature. It is interesting to note that the values obtained by Furnas (*8*) (curve *K*), in which a pseudo steady state method of determining the coefficients was used, are in almost exact agreement with those of the authors. The data of Gamson (curve *G*) were obtained from experiments on the drying of wet catalyst carriers during the constant-rate drying period, and the coefficients were calculated on the assumption that adiabatic conditions prevailed in the bed. The bed was quite shallow in comparison with the particle size, and it was assumed that the entire surface of the pellets was wet. The curves of Denton (curves *HIJ*) are the results of experiments carried out by placing test spheres at various points in a packed bed. Heat was generated in the test spheres by means of a resistance heater inside each sphere. The experiments of Dyankonov (curves *EF*), which were published after the present work was under way, were carried out in approximately in the same way as those of this work, induction heating being used to generate heat in the pellets. The average temperature differences at the entrance and exit of the bed were taken as an arithmetic average of several values across the diameter of the bed. These values were not reported, but it was found in the present investigation that the temperature differences near the tube wall were much higher than those near the center of the bed, because of uneven heat generation in the bed. Therefore the actual mean difference would be somewhat higher than the arithmetic average difference. The flow rates and heat fluxes used for these experiments were much lower than those of the authors.

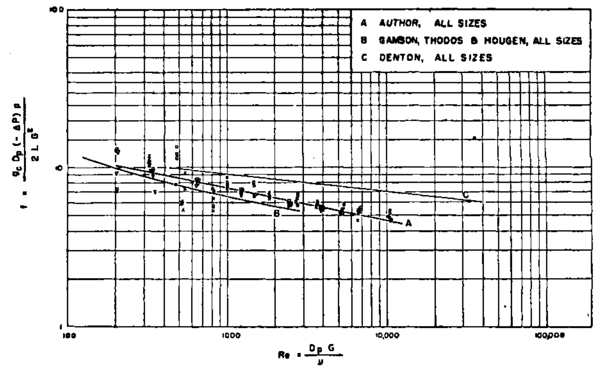

Fig. 5. Correlation of friction factor with Reynolds number.

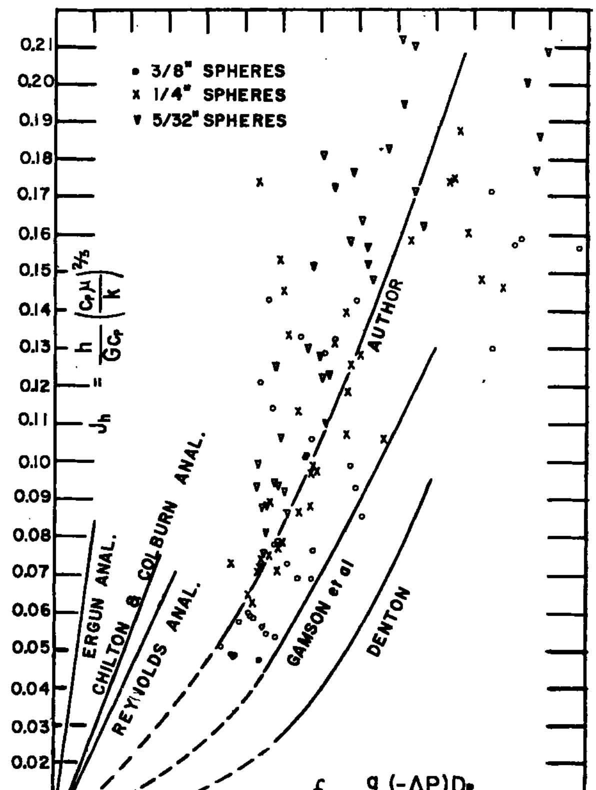

Fig. 6. Heat-transfer and pressure-drop analogies.

EFFECT OF TUBE- TO PARTICLE-DIAMETER RATIO

In correlating the heat transfer coefficients, it was found that the coefficient of the Reynolds number as well as the

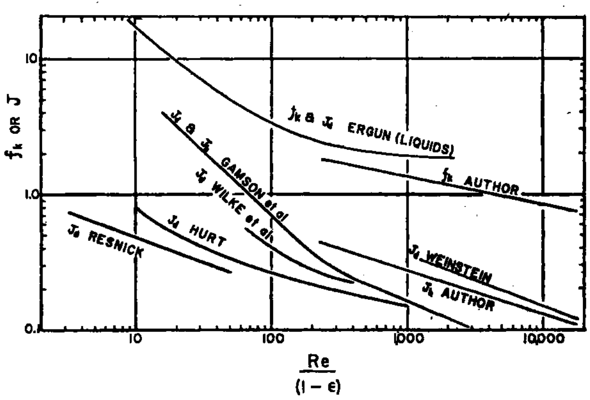

Fig. 7. Comparison of data for liquids and gases.

exponent varied with tube to particle diameter. Figure 4 shows the variation obtained from this experiment for the j_h correlation. The dotted line represents the data obtained by Denton. The following expressions were obtained:

$$a = 0.918[1 + 0.0148e^{0.565(18 - D/Dp)}] \quad (2)$$

$$b = -0.267 - \frac{0.257}{D/Dp - 8.70} \quad (3)$$

From these equations it can be seen that the effect of the ratio on the coefficient of the Reynolds number, a, becomes negligible above a value of about 18. The exponent, b, declines less rapidly as the ratio increases, but it can be seen that the effect of the ratio is still noticeable even at the highest ratios. These variations are due to the distortion in the packing at the tube wall. This wall effect is great for the smaller values of the ratio and diminishes as the ratio increases.

HEAT TRANSFER AND PRESSURE-DROP ANALOGIES

Figure 5 shows the pressure-drop data obtained in this investigation correlated in terms of Fanning friction factors. The data are represented by the equation

$$f = 30.7\, Re^{-0.20} \quad (4)$$

According to the analogies developed by Reynolds and by Chilton and Colburn for straight ducts, j_h could be expressed as a linear function of the friction factor. For packed beds these analogies can be expressed as

$$j_h = (Pr)^{2/3} \frac{\epsilon^3}{3(1 - \epsilon)} f \quad (5)$$

for the Reynolds analogy and

$$j_h = \frac{\epsilon^3}{3(1 - \epsilon)} f \quad (6)$$

for the Chilton and Colburn analogy. These equations are shown in graphical form in Figure 6 along with the data of this experiment. It can be seen that these analogies do not hold for this case and that the relationship between j_h and f is not linear as predicted. From the empirical equations developed for both, the relationship was found to be

$$j_h = 1.33\, Re^{-0.12} \frac{\epsilon^3}{3(1 - \epsilon)} f \quad (7)$$

However, these analogies were developed for fluids flowing in straight ducts and would therefore not be expected to represent the data for a packed bed.

In correlating pressure-drop measurements from several investigations involving liquids, Ergun (6) described a friction factor which he defined as

$$f_k = \frac{(-\Delta P)}{L} \frac{g_c D_p}{\rho U^2} \frac{\epsilon^3}{(1 - \epsilon)} \quad (8)$$

In another article (7) Ergun correlated mass transfer data obtained by a number of investigators and developed an analogy for packed beds which fit the data as follows:

$$J_d = f_k = 1.75 + 150 \frac{(1 - \epsilon)}{Re} \quad (9)$$

where

$$J_d = \frac{6\epsilon k_g M_m}{G} \left(\frac{\mu}{D_f \rho}\right) \quad (10)$$

or for heat transfer this corresponds to

$$J_k = 6 \frac{\epsilon h}{G C_p} \left(\frac{C_p \mu}{k}\right) \quad (11)$$

This analogy is shown in Figure 7 along with the results of the authors as well as mass transfer data of other investigations.

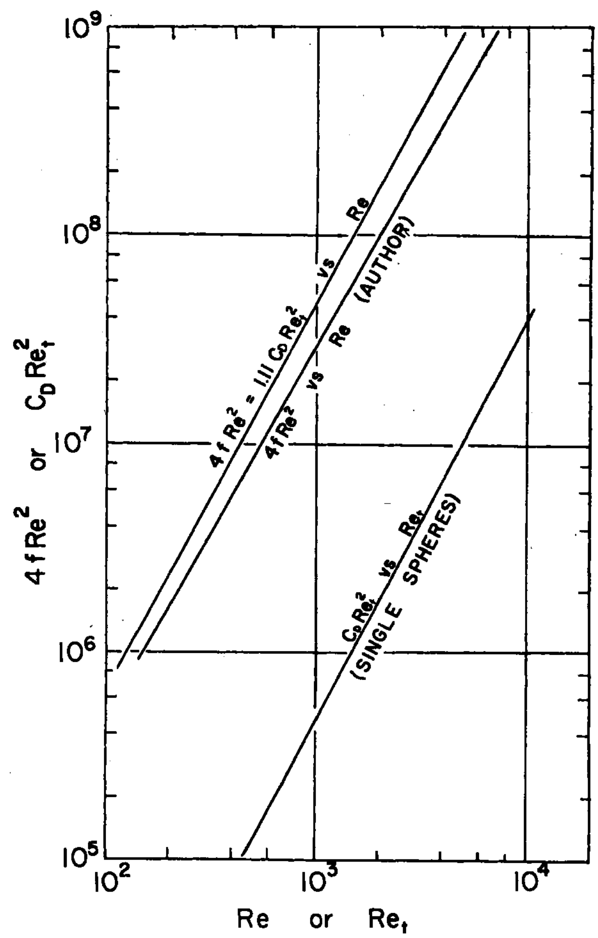

Fig. 8. Frictional resistance for single spheres and packed beds.

It can be seen that whereas this analogy works for liquids, the data for gases are quite different.

PREDICTION OF PACKED-BED TRANSFER RATES FROM RATES FOR SINGLE PARTICLES

It has been found that most of the pressure drop due to the flow around a single sphere takes place on the forward face of the sphere. Therefore, since the points of contact in a packed bed are at points of low pressure drop and the front faces of the spheres receive the full fluid stream, it is plausible that the pressure drop through a packed bed is an additive property of the pressure drop around single spheres. By assuming a model packing in a rhombohedral arrangement, Ranz (*14*) developed the following relation:

$$4f\,Re^2 = 1.111 C_D\,Re'^2$$

where Re′ is the Reynolds number based on the actual velocity at the front faces of the spheres in the bed. From the geometry of the packing it was calculated that

$$Re'/Re = 10.73$$

It has also been found that the heat and mass transfer rates around a sphere are at a maximum on the front face of the sphere, and therefore Ranz suggested that the transfer rates in packed beds would be the same as the transfer rates around a single sphere acted upon by a fluid stream with a velocity 10.73 times as great as the superficial velocity. Since this type of packing is idealized, it was suggested that for an actual bed the parameter to be used in predicting the transfer rates for the bed would be the ratio of Re'/Re as determined from pressure-drop data.

Figure 8 shows a plot of the pressure-drop data obtained in this investigation as well as the data for single spheres. It can be seen that the friction losses predicted by the model bed are slightly higher than those found by the authors, but they are in good agreement considering the crude model that was used for the mathematical development. From the data of this investigation, the parameter Re'/Re was found to be 9.4. Figure 9 shows the heat transfer rates for single spheres (*12*) and also the rates which would be predicted from this parameter in terms of Nusselt numbers. The predicted rates are somewhat lower than the data obtained in this investigation. The agreement is better near the low Reynolds number range, but it can be seen from the data that the rate of heat transfer from a single sphere is not affected so much by the Reynolds number as is that from the spheres in the packed bed.

SUMMARY

The average heat transfer coefficients for a packed bed vary with the ratio of tube to particle diameter for a given Reynolds number. The effect is quite significant for small values of this ratio and diminishes as the ratio is increased. Equations were developed which can be used to calculate the average heat transfer coefficient for a packed bed in terms of both the Reynolds number and the ratio of tube to particle diameter.

The analogies advanced for the simple interrelation of pressure drop, heat transfer, and mass transfer do not apply satisfactorily for packed beds when gas is used as the fluid.

By the method of Ranz, it was possible to predict the pressure drop for a packed bed fairly accurately. When the parameter suggested by this method was used, the predicted heat transfer coefficients were appreciably lower than the experimental values of this investigation.

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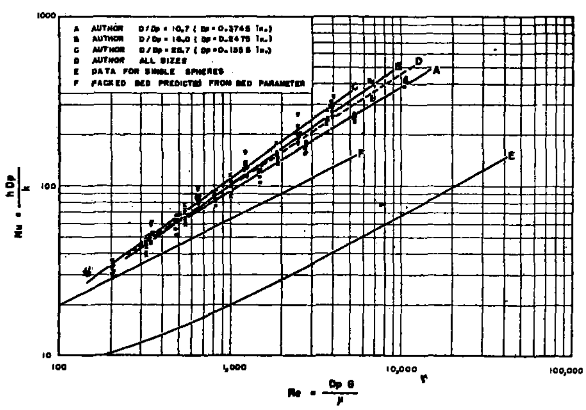

Fig. 9. Correlation of Nusselt number with Reynolds number.

NOTATION

a = constant of proportionality
b = exponent for j_h correlation
C_p = fluid heat capacity
D = tube diameter
D_f = molecular diffusivity
D_p = particle diameter
f = Fanning friction factor
f_k = Ergun friction factor
G = mass velocity
g_c = conversion factor
h = average heat transfer coefficient
j = Colburn analogy factor
J = Ergun analogy factor
k = fluid thermal conductivity
k_g = mass transfer coefficient
M_m = mean molecular weight
P = Pressure
Pr = Prandtl number
Re = Reynolds number based on superficial velocity
Re' = Re based on velocity through cross section open to flow
U = superficial velocity

Greek Letters

ϵ = void fraction
μ = viscosity
ρ = fluid density

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